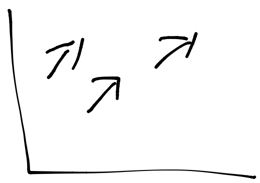
Euler's method

$z_{n+1} = z_n + f(t_n, z_n)\,\Delta t$

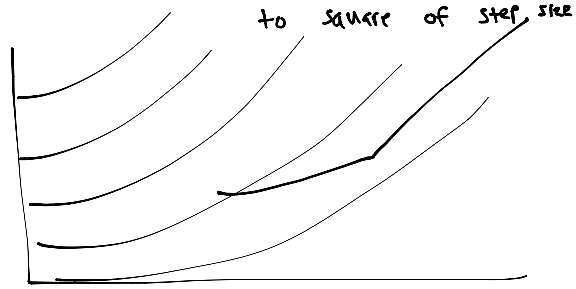
What is an ode?
- vector field



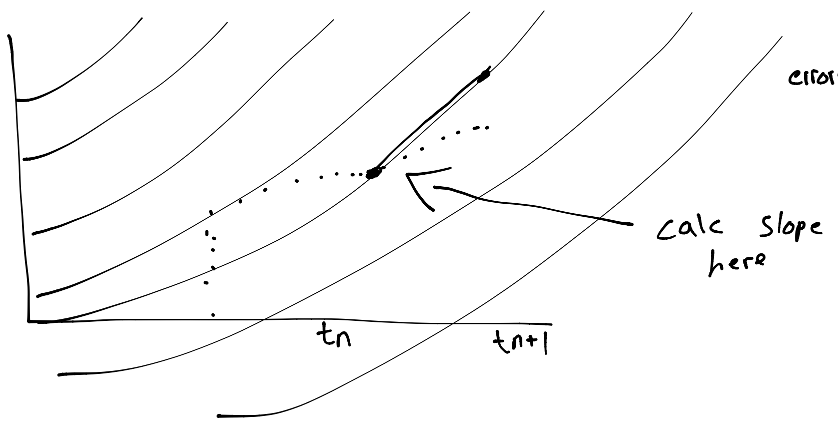
$\dot{z} = f(t, z)$

Solution is tangent to arrows

error is proportional to square of step size



Midpoint Method RK2



errors $\approx h^3$

$z_{temp} = z_n + f(t_n, z_n)\,\frac{h}{2}$

$z_{n+1} = z_n + f(t_n + \frac{h}{2}, z_{temp})\,h$

rhs fn

Work and Energy

$$\boxed{\begin{array}{ll} \int(\vec{F}=m\vec{a})\,dt \rightarrow \text{Impulse Momentum} \\ \vec{r}\times\vec{F}=m\vec{a} \rightarrow \text{angular momentum} \end{array}}$$

Last time

Moving On:

$\vec{F}=m\vec{a}$

$\vec{F}\cdot\vec{v} = m\vec{a}\cdot\vec{v}$ We observe that: $\frac{d}{dt}(v^2)=\frac{d}{dt}\vec{v}\cdot\vec{v}$

$$\frac{d}{dt}(v^2)=\frac{d}{dt}\vec{v}\cdot\vec{v} = \dot{\vec{v}}\cdot\vec{v}+\vec{v}\cdot\dot{\vec{v}} = 2\dot{\vec{v}}\cdot\vec{v} = 2\vec{a}\cdot\vec{v}$$

$$\boxed{\underbrace{\vec{F}\cdot\vec{v}}_{\text{power}} = \frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}mv^2}_{E_K:\ \text{Kinetic Energy}}\Big) \qquad P=\dot{E}_K}$$

$$\left(\int P = \int \dot{E}_K\right)dt \longrightarrow \boxed{\int_{t_1}^{t_2} P\,dt = \Delta E_K = E_{K_2}-E_{K_1}}$$

$$\int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2}\vec{F}\cdot\underline{\vec{v}\,dt} \longrightarrow \vec{v}\,dt = d\vec{r}$$

$$\boxed{W = \int_{r_1}^{r_2}\vec{F}\cdot d\vec{r}\,,\quad \boxed{W=\Delta E_K}} \longrightarrow \sum \vec{F}\cdot\Delta\vec{r}$$

path integral

Reimann Sum

special cases: Conservative forces

$\vec{F}(t,\vec{r},\vec{v}) = \vec{F}(\vec{r})$ and they are conservative

- depend on position only

Conservative: Some scalar field exists $V(x,y) = V(\vec{r})$

$\vec{F} = -\vec{\nabla} V$ $\qquad V = E_p =$ potential energy

$F_x = \dfrac{-\partial E_p}{\partial x}$, $F_y = \dfrac{-\partial E_p}{\partial y}$ $\qquad$ if true, $\vec{F}$ is conservative

<u>Example:</u> near Earth gravity:

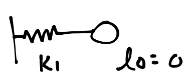
$\vec{F} = -mg\hat{j}$ $\qquad F_x = 0,\ F_y = -mg$ $\qquad E_p = mgy \ + \underbrace{[C]}_{\text{no need to write}}$ (constant)

Check: $\dfrac{-\partial E_p}{\partial x} = \dfrac{-\partial mgy}{\partial x} = 0 \checkmark$

$\dfrac{-\partial E_p}{\partial y} = \dfrac{-\partial mgy}{\partial y} = -mg \checkmark$

$\Big\}$ conservative

<u>Example:</u> zero-rest-length spring

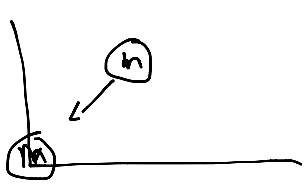


$\vec{F} = -k\vec{r}$

$E_p = \frac{1}{2}kr^2$

check: $-\vec{\nabla} E_p = -k\overbrace{r\hat{e}_r}^{\vec{r}}$ $\checkmark$

<u>Example:</u> Inverse Square Gravity



$\vec{F} = \dfrac{-mMG}{r^2}\hat{e}_r$ $\qquad$ only a function of r

$E_p = -\dfrac{mMG}{r}$

$$-\vec{\nabla} E_p = -+\left(\frac{mMG}{r^2}\right)\hat{e}_r \qquad -\nabla E_p = \frac{-mMG}{r^2}\hat{e}_r \checkmark$$

Example: $\vec{F} = y\hat{i} - x\hat{j}$

$$\int \vec{F} = \int y\hat{i} - x\hat{j} \qquad F_x = \frac{-\partial E_p}{\partial x} \qquad F_y = \frac{-\partial E_p}{\partial y}$$

$$\frac{\partial F_x}{\partial y} = -1 \qquad \frac{\partial F_y}{x} = 1$$

$$\frac{\partial^2 E_p}{\partial y \partial x} \neq \frac{\partial^2 E_p}{\partial x \partial y}$$

No such scalar exists: $\vec{F}$ is not conservative